## Inductive and Deductive Reasoning

Name ____________________

### General Outcome
Develop algebraic and graphical reasoning through the study of relations

### Specific Outcomes
it is expected that students will:

<table>
<thead>
<tr>
<th>11a.l.1.</th>
<th>Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>11a.l.2.</td>
<td>Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.</td>
</tr>
</tbody>
</table>

### Achievement Indicators
The following set of indicators may be used to determine whether students have met the corresponding specific outcome

<table>
<thead>
<tr>
<th>11a.l.1.</th>
<th>(it is intended that this outcome be integrated throughout the course)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Make conjectures by observing patterns and identifying properties, and justify the reasoning.</td>
</tr>
<tr>
<td></td>
<td>Explain why inductive reasoning may lead to a false conjecture.</td>
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<tr>
<td></td>
<td>Compare, using examples, inductive and deductive reasoning.</td>
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<tr>
<td></td>
<td>Provide and explain a counterexample to disprove a conjecture.</td>
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<tr>
<td></td>
<td>Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies, or algebraic number tricks.</td>
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<tr>
<td></td>
<td>Prove a conjecture, using deductive reasoning (not limited to two column proofs).</td>
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<tr>
<td></td>
<td>Determine if an argument is valid, and justify the reasoning.</td>
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<tr>
<td></td>
<td>Identify errors in a proof.</td>
</tr>
<tr>
<td></td>
<td>Solve a contextual problem involving inductive or deductive reasoning.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>11a.l.2.</th>
<th>(it is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Determine, explain and verify a strategy to solve a puzzle or to win a game such as</td>
</tr>
<tr>
<td></td>
<td>• guess and check</td>
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<tr>
<td></td>
<td>• look for a pattern</td>
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<tr>
<td></td>
<td>• make a systematic list</td>
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<td></td>
<td>• draw or model</td>
</tr>
<tr>
<td></td>
<td>• eliminate possibilities</td>
</tr>
<tr>
<td></td>
<td>• simplify the original problem</td>
</tr>
<tr>
<td></td>
<td>• work backward</td>
</tr>
<tr>
<td></td>
<td>• develop alternative approaches</td>
</tr>
<tr>
<td></td>
<td>Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.</td>
</tr>
<tr>
<td></td>
<td>Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.</td>
</tr>
</tbody>
</table>
Exercise 1: Making Conjectures: Inductive Reasoning

Conjecture:

Inductive Reasoning:

Make a conjecture about diagonal $AB$ and diagonal $BC$.

Make a conjecture about the circles in the centre.

Make a conjecture about the number of triangles.

Make a conjecture about the lines.

Inductive Reasoning:
Weather Conjectures

Long before weather forecasts based on weather stations and satellites were developed, people had to rely on patterns identified from observation of the environment to make predictions about the weather. (From Nelson Foundations of Math.)

For example:

- **Animal behaviour**: First Nations peoples predicted spring by watching for migratory birds. If smaller birds are spotted, it is a sign that spring is right around the corner. When the crow is spotted, it is a sign that winter is nearly over. Seagulls tend to stop flying and take refuge at the coast when a storm is coming. Turtles often search for higher ground when they expect a large amount of rain. (Turtles are more likely to be seen on roads as much as 1 to 2 days before rain.)

- **Personal**: Many people can feel humidity, especially in their hair (it curls up and gets frizzy). High humidity tends to precede heavy rain.

For Example:

Examine the pattern below. Make a prediction about the next numbers.

\[ 1^2 = 1 \]
\[ 101^2 = 10201 \]
\[ 10101^2 = 102030201 \]
\[ 1010101^2 = 1020304030201 \]

\[ \square = \square \]
\[ \square = \square \]

Conjecture: ____________________________________________
For Example:

Examine the pattern in the addends and their sums. What conjecture can you make?

\[
\begin{align*}
1 + 3 &= 4 \\
3 + 5 &= 8 \\
5 + 7 &= 12 \\
7 + 9 &= 16
\end{align*}
\]

Conjecture: _______________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

Much of the reasoning in geometry consists of three stages:

1. Look for a pattern by using several examples to see whether you can discover the pattern.

2. Make a generalization using several examples. The generalization is called a conjecture. You then check with more examples to confirm or refute the conjecture. These first two steps involve inductive reasoning to form the conjecture.

3. Verify that your conjecture is true in all cases by using logical reasoning.
For Example:

Draw a pair of intersecting lines. Measure and record the opposite angles as shown. What conjecture can you make?

Conjecture: __________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________

For Example:

Given the following diagram, can you make a conjecture about the number of triangles?

Conjecture: __________________________________________________________
_______________________________________________________________
_______________________________________________________________
_______________________________________________________________
For Example:

- If 2 points are marked on the circumference of a circle, they can be joined to form 1 chord which will divide the circle into two regions.
- If you have three points, you get 3 chords which gives four regions.
- Four points make 6 chords, and 8 regions and so on.
- Use inductive reasoning to come up with a conjecture about the number of points and the chords that can be made.

Solution:

<table>
<thead>
<tr>
<th>Number of Points</th>
<th>Number of Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
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</tbody>
</table>

Conjecture: __________________________________________

_____________________________________________________

_____________________________________________________
Exercise 2: Exploring the Validity of Conjectures

For Example:

Are the horizontal lines in this diagram straight or curved? Make a conjecture. Then, check the validity of your conjecture.

Solution:

For Example:

Make a conjecture about this pattern. How can you check the validity of your conjecture?

\[
\frac{1}{1} + \frac{1}{2} = \frac{3}{2}, \quad \frac{1}{2} + \frac{1}{3} = \frac{5}{6}, \quad \frac{1}{3} + \frac{1}{4} = \frac{7}{12}, \quad \frac{1}{5} + \frac{1}{6} = \frac{11}{30}
\]

Solution:
Exercise 3: Using Reasoning to Find a Counterexample to a Conjecture

Counterexample:_________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

For Example:

Examine this pattern in the product of 11 and another two-digit number.

\[11 \times 11 = 121\]
\[12 \times 11 = 132\]
\[26 \times 11 = 286\]
\[43 \times 11 = 473\]

What conjecture can you make about the pattern?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

How can you check whether your conjecture is true?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

Can you come up with a new conjecture?
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
Remember the Example:

- If 2 points are marked on the circumference of a circle, they can be joined to form 1 chord which will divide the circle into two regions.
- If you have three points, you get 3 chords which gives four regions.
- Four points make 6 chords, and 8 regions and so on.

- Use inductive reasoning to come up with a conjecture about the number of points and the chords that can be made.

Solution:

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Conjecture: As the number of points on the circle increases, the number of regions increases by a factor of 2. Let’s carefully try 6 points...
For Example:

Jessica stated that: \( a^b = b^a. \)

a) Find an example to show that her conjecture is reasonable.

b) Find a counterexample to prove that the conjecture is false.

For Example:

Ryun concluded that whenever he added two prime numbers, the sum was always even. Find a counterexample to prove that Ryun’s conjecture is false.

Solution:

For Example:

Give a counterexample to disprove the following statements:

a) All prime numbers are odd.

b) A quadrilateral with a pair of parallel sides is a parallelogram.

c) If four vertices of a quadrilateral lie on the same circle, then the quadrilateral is a parallelogram.
Exercise 4: Proving Conjectures: Deductive Reasoning

Proof:

_____________________________________________________________________

_____________________________________________________________________

Generalization:

_____________________________________________________________________

_____________________________________________________________________

Deductive Reasoning:

_____________________________________________________________________

_____________________________________________________________________

Transitive Property:

_____________________________________________________________________

_____________________________________________________________________

For Example:

Use deductive reasoning to make a conclusion from these statements:

- "All koalas are marsupials. All marsupials are mammals."

- "All mammals are warm-blooded. Barney is a koala."

Solution:
For Example:

Use deductive reasoning to prove this conjecture:

- "The difference of two perfect squares that are separated by one perfect square is always even."

Solution:

For Example:

Show deductively that the sum of the measures \(a\), \(b\), and \(c\) is 180°.

Solution:

\[
\begin{align*}
a) & \quad \text{Draw a triangle.} \\
b) & \quad \text{Use one side as a base and draw a parallel line segment on the opposite vertex.} \\
c) & \quad \text{Use the known fact that } a = a, b = b, \text{ and } c \text{ is included in both, therefore } a + c + b = 180°. \\
\end{align*}
\]
For Example:

All Vincent Massey students are cool.
James is a Vincent Massey student.

Therefore, ________________________________

For Example:

Anyone who likes to play badminton likes to play tennis.
Jeff likes to play badminton.

Therefore, ________________________________

For Example:

All students in Senior Years must enroll in English.
Tracy is a student.

Therefore, ________________________________
Inductive or Deductive? Which of the conclusions below are valid?

a) The sun will rise tomorrow?

b) All Vincent Massey students love their calculators.
   \( \therefore \) Anyone who hates their calculator must not be a Vincent Massey student.

c) There will be snow at Christmas time.

d) \( 3x = 15, \ \therefore x = 5 \)

e) There will be at least one big flood in Winnipeg every 100 years.

f) The radius of a circle is 5cm. The area of that circle is \( 25\pi \) cm\(^2\).

g) There is a full moon once a month.

h) Every time I have a hockey game, I have a test the following day.

i) During a statistics project, Jeanine counted the number of cars of different colours that passed her school in 20 minutes. More than half the cars were red. She decided that red is the most popular colour for cars.

j) Triangle ABC is an equilateral triangle.
   We conclude that \( AB \cong AC \).
Exercise 5: Proofs That Are Not Valid

Proving Something Invalid:__________________________________________________
_____________________________________________________________________
_____________________________________________________________________
                                                   ______________________
Premise:_______________________________________________________________

Circular Reasoning:_____________________________________________________
                                                                         ______________________

For Example:

What type of error occurs in this deduction?

• All people over 65 are retired.
• Corbin is over 65, so he is retired.

For Example:

What type of error occurs in this deduction?

\[ 6 = 6 \]

\[ 2.5(6) = 2.5(3 + 3) \]

\[ 2.5(6) + 1 = 2.5(3 + 3) + 1 \]

\[ 15 + 1 = 7.5 + 4 \]

\[ 16 = 11.5 \]
For Example:
What type of error occurs in this deduction?

Mr. A. "Do you believe in god?"
Mr. B. "YES"
Mr. A. "Why do you believe in god?"
Mr. B. Because it is written in the Bible.
Mr. A. "Why do you believe the Bible?"
Mr. B. "Because the Bible is the word of god"

For Example:
What type of error occurs in this deduction?

Let: \( a = b = 1 \)
To prove: \( 2 = 0 \)

Proof:
\[ \begin{align*}
    a &= b \\
    a^2 &= b^2 & \text{Same operation on both sides} \\
    a^2 - b^2 &= 0 & \text{Move } b^2 \text{ to the other side} \\
    (a - b)(a + b) &= 0 & \text{Factor a Difference of Squares} \\
    \frac{(a - b)(a + b)}{(a - b)} &= \frac{0}{(a - b)} & \text{Same operation on both sides} \\
    (a + b) &= 0 & \text{Simplify} \\
\end{align*} \]

But: \( a = b = 1 \)

Therefore: \( 1 + 1 = 0 \) \hspace{1cm} \text{Substitution}
\[ 2 = 0 \]
For Example:

What type of error occurs in this deduction?

13 \times 5 = 65

8 \times 8 = 64

Therefore, if the shapes are all the same, ...

65 = 64
Exercise 6: Reasoning to Solve Problems

For Example:

Ten cards, numbered from 0 to 9, are divided among five envelopes, with two cards in each envelope. Each envelope shows the sum of the cards inside it.

- The envelope marked 10 contains the 6 card.
- The envelope marked 14 contains the 5 card.

What pairs of cards does each envelope contain? Explain.

Solution:

5  7  9  10  14
For Example:

Six friends are sitting around a circular table in a noisy restaurant. Only two friends sitting next to each other can have a conversation. The friends decide to change seats at various times during the evening to make sure everyone gets a chance to talk to everyone else. What is the least number of times the friends have to change seats?

Solution:
Exercise 7: Analyzing Puzzles and Games

For Example:

What numbers go in squares A and B of this Sudoku puzzle?

To solve a Sudoku puzzle:

- Fill in each empty square with a number from 1 to 9.

- A number cannot appear twice in a column, row, or 3 X 3 block square.

```
   9  6
   1
   2
   4

   5
   7
   3
   4

   1
   2
   7
   3

   4
   5
   9
   1

   2
   4
   5
   9

   6
   5
   8

   B
   6
```
For Example:

Solve this Kakuro puzzle:

- Fill in each empty square with a number from 1 to 9.
- Each row must add up to the circled number on the left.
- Each column must add up to the circled number above.
- A number cannot appear twice in the same row or column.
For Example:

In the Prisoner’s Dilemma, two prisoners are being held by the authorities in separate interrogation rooms. Each prisoner has the same options: he or she can cooperate with the other prisoner by remaining silent, or defect, confessing to the authorities. The results of cooperating or defecting depend on what both prisoners do.

- If both prisoners cooperate, they are each sentenced to 1 year in prison.
- If both prisoners defect, they are each sentenced to 3 years.
- If one prisoner cooperates and the other defects, the defector is sentenced to 6 months and the cooperative prisoner is sentenced to 10 years.

a) What would be the best thing for the prisoners to do, and what sentence would they get, if each knew the others decision?

b) What do you think the prisoners will actually do, and what sentence will they get, given that they do not know the other's decision?
For Example:

Fill in the missing numbers, from 1 to 9, so that the sum of the numbers in each row, column, and diagonal is 15.

```
  |   | 6 |
---|---|---|
  | 1 |
  | 4 |
  | 3 |
  | 8 |
```

```
  |   | 4 |
---|---|---|
  | 5 |
  | 3 |
  | 8 |
```

For Example:

Sudoku Puzzles - In the first, use the numbers from 1 to 6, in the second, 1 to 9. Each number must appear in every row, column and block.

```
 5 | 2 | 6 |
---|---|---|
 4 |
 1 | 6 |
 1 | 5 |
 3 | 2 | 1 |
---|---|---|
 6 |
```

```
 6 | 4 | 8 | 2 |
---|---|---|---|
 1 | 4 |
 6 | 3 | 5 |
 1 | 4 |
 8 | 9 | 2 |
---|---|---|
 1 | 3 | 2 | 9 |
 4 | 5 | 6 |
 5 |
```

**For Example:**

KenKen: Use only the numbers from 1 to 6. Each number must appear in every row and column. A number may be repeated in a cage (dark outlines), but not a row or column. Use numbers which give the target in each cage by using the given operation.

<table>
<thead>
<tr>
<th></th>
<th>30 $\times$</th>
<th>36 $\times$</th>
<th>2 $\div$</th>
<th>18 $+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5?</td>
<td>6?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 +</td>
<td></td>
<td></td>
<td>7 +</td>
<td></td>
</tr>
<tr>
<td>20 $\times$</td>
<td></td>
<td>5 $-$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 $-$</td>
<td>2 $-$</td>
<td>13 $+$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 +</td>
<td>2 $-$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 $\div$</td>
<td></td>
<td>3 $-$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 and 6 give a product of 30.
For Example:

How many ways can the mouse navigate the maze to reach the trail mix, if the mouse can only travel down?
Inductive and Deductive Reasoning - CHAPTER 1 TEST

Name_________________________ Date___________________

1. Hilary was examining the differences between perfect squares of numbers separated by 5. She made the following conjecture: The differences always have the digit 5 in the ones place.

For example:

\[ 17^2 - 12^2 = 289 - 144 = 145 \]

a) Gather evidence to support Hilary's conjecture.

b) Is her conjecture reasonable? Explain.

2. Denyse works part time at a grocery store. She notices that the store is very busy when she works in the afternoon from 4 to 7 p.m., but it is less busy when she works in the evening from 7 to 10 p.m.

What conjecture can you make for this situation? Justify your conjecture.
3. Heather claimed that the sum of two multiples of 4 is a multiple of 8.

Is Heather's conjecture reasonable? Explain. If it is not reasonable, find a counterexample.

4. Prove that the sum of two consecutive perfect squares is always an odd number.

5. Prove that the following number trick will always result in 6:
   · Choose any number.
   · Add 3.
   · Multiply by 2.
   · Add 6.
   · Divide by 2.
   · Subtract your starting number.
6. Judd presented the following argument:

Inuvik, Northwest Territories, is above the Arctic Circle, which is at a latitude of 66° north of the equator. Places north of the Arctic Circle have cold, snowy winters. Winnipeg is at a latitude of 52° north of the equator. Therefore, Winnipeg does not have cold, snowy winters.

Is Judd’s argument reasonable? If not, identify the errors in his reasoning.

7. Is this proof valid? Explain.

<table>
<thead>
<tr>
<th>Let any number, a, equal b.</th>
<th>Multiply both sides by a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 = ab$</td>
<td></td>
</tr>
<tr>
<td>$a^2 - b^2 = ab - b^2$</td>
<td>Subtract $b^2$ from both sides.</td>
</tr>
<tr>
<td>$(a + b)(a - b) = b(a - b)$</td>
<td>Factor both sides.</td>
</tr>
<tr>
<td>$a + b = b$</td>
<td>Divide by $(a - b)$.</td>
</tr>
<tr>
<td>$a + b - b = b - b$</td>
<td>Subtract $b$ from both sides.</td>
</tr>
<tr>
<td>$a = 0$</td>
<td>Any number equals zero.</td>
</tr>
</tbody>
</table>

8. Fill in the circles with the appropriate numbers to make the puzzle work.

a) \[17 + \quad \quad \quad + \quad \quad \quad 18\]

b) \[9 + \quad \quad \quad 12\]

c) \[38 + \quad \quad \quad 48\]