Systems of Linear Inequalities

Name ____________________________
Graph and Solve linear inequalities in one dimension

Single-Variable Inequality Notations

There are 3 different types of single-variable notation:

I. Set Notation
II. Number Line Notation
III. Interval Notation

I. Set Notation:

You will want to remember what each inequality symbol means. This will be easier to do if you remember that the open part of the symbol always faces the larger quantity.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
<td>less than</td>
</tr>
<tr>
<td>&gt;</td>
<td>greater than</td>
</tr>
<tr>
<td>&lt;=</td>
<td>less than or equal to</td>
</tr>
<tr>
<td>&gt;=</td>
<td>greater than or equal</td>
</tr>
</tbody>
</table>

For example,

\[ x < 7 \quad \text{x is less than seven.} \]

\[ x < 7 \quad \text{Seven is greater than x.} \]

Sometimes it may be required to solve an inequality (i.e. isolate x).

First, let’s review solving equations (with an equal sign).

Examine the method used to solve the following equation:

\[ 2x - 5 = 9 \]

\[ 2x = 14 \quad \text{add 5 to both sides} \]

\[ x = 7 \quad \text{divide both sides by 2} \]

The solution to this equation is exactly 7.
Now examine the method used to solve the following inequality:

\[2x - 5 < 9\]

\[
\begin{align*}
2x &< 14 & \text{add 5 to both sides} \\
x &< 7 & \text{divide both sides by 2}
\end{align*}
\]

The solution to this inequality is all real numbers less than 7.

**Solving Inequalities**

**Example 1**

Solve \[3x - 5 < -14\]

\[
\begin{align*}
3x &< -9 & \text{add 5 to both sides} \\
x &< -3 & \text{divide both sides by 3}
\end{align*}
\]

The solution to this inequality is all real numbers less than -3.

Solving linear inequalities is very much the same as solving linear equations with one very important exception!

If while solving an inequality a negative number is used to multiply or divide each side of the inequality then the sense of the inequality must be reversed.

**When multiplying or dividing by a negative number, the inequality sign flips!**
Applied Math 30S

Example 2

\[4x - 1 \leq 6x - 7\]

\[4x - 6x \leq -7 + 1\]

\[-2x \leq -6\]

\[\frac{-2}{-2} x \geq \frac{-6}{-2}\]

\[x \geq 3\]

*The solution to this inequality is all real numbers greater than or equal to 3.*

Since the left and right sides of the inequality were divided by -2, the inequality sign must flip (from \(\leq\) to \(\geq\)).

So far all of the solutions had intervals bounded by integers. The boundary does not have to be an integer. Here is a solved example of a boundary that is a rational number.

*Example 3*

\[3x - 2 > 11\]

\[3x > 11 + 2\]

\[3x > 13\]

\[\frac{3}{3} x > \frac{13}{3}\]

\[x > \frac{13}{3}\]

*The solution to this inequality is all real numbers greater than \(\frac{13}{3}\).*

*Leave the final answer in set notation as a fraction, rather than approximating with decimals (i.e. dividing and rounding).*

II. **Number Line Notation:**
The expression $x < 7$ can be represented by the number line graph shown below. **Note: the OPEN circle since 7 is NOT included in the solution.**

The expression $x \leq 7$ (less than or equal to 7) would be represented by the number line below. **Note: the FILLED circle since 7 is included in the solution.**

**Number Line Summary:**

<table>
<thead>
<tr>
<th>Less than</th>
<th>Greater than</th>
<th>Equal to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrow left</td>
<td>Arrow right</td>
<td>Closed circle</td>
</tr>
<tr>
<td>Not equal to</td>
<td>Open circle</td>
<td></td>
</tr>
</tbody>
</table>

**III. Interval Notation:**

The expression $x < 7$ can be represented by the interval notation shown below. **Note: this notation represents all the real numbers from negative infinity to 7 (but not including 7).**

$(-\infty, 7)$

The expression $x \leq 7$ (less than or equal to 7) would be represented by the interval notation shown below. **Note: To include the 7 use a square bracket around 7.**

$(-\infty, 7]$
The intervals can also be described in words.

- ( ) : open interval
- [ ] : closed interval
- ( ] and [ ) : half open interval.

**Notations: Sample Questions with Solutions**

Given the following set notation, determine the number line and the interval notation to match.

<table>
<thead>
<tr>
<th>Set Notation</th>
<th>Number Line Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &gt; -2 )</td>
<td><img src="image1" alt="Number Line 1" /></td>
<td>((-2, \infty))</td>
</tr>
<tr>
<td>( x \geq 0 )</td>
<td><img src="image2" alt="Number Line 2" /></td>
<td>([0, \infty))</td>
</tr>
<tr>
<td>( 3 &lt; x &lt; 7 )</td>
<td><img src="image3" alt="Number Line 3" /></td>
<td>((3, 7))</td>
</tr>
<tr>
<td>( -1 &lt; x \leq 4 )</td>
<td><img src="image4" alt="Number Line 4" /></td>
<td>((-1, 4])</td>
</tr>
</tbody>
</table>

**Notation Summary Review:**

<table>
<thead>
<tr>
<th>Inequalities</th>
<th>Set Notation</th>
<th>Number Line</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not equal (not included)</td>
<td>&lt; and &gt;</td>
<td>open circle</td>
<td>open ( ) bracket</td>
</tr>
<tr>
<td>Equal to (included)</td>
<td>( \leq ) and ( \geq )</td>
<td>filled circle</td>
<td>closed [ ] bracket</td>
</tr>
</tbody>
</table>

Note: Infinity (\(\infty\)) always takes an open bracket.
**Review: Intercepts Graphing Method**

If the problem is given in standard form

\[ Ax + By = C \]

**Sample Question**

Graph \( 2x + 3y = 6 \) using the Intercepts Graphing Method.

**Sample Solution**

Find \( x \)

Step 1: cover \( y \) term

\[ 2x + 3y = 6 \]

Step 2: solve for \( x \)

\[ 2x = 6 \]  
(divide 6 by 2)  
\[ x = 3 \]  
*this will be plotted on the \( x \)-axis

Find \( y \)

Step 3: cover \( x \) term

Step 4: solve for \( y \)

\[ 3y = 6 \]  
(divide 6 by 3)  
\[ y = 2 \]  
*this will be plotted on the \( y \)-axis

Step 5: Plot the values (intercepts) on each axis and connect them with a straight line.

Remember!
- Label lines with equation
- Use a ruler
- Use arrows
- Long lines across the graph
Graphing the line of inequalities is similar to graphing the line of equations with TWO EXCEPTIONS.

**EXCEPTION 1**
A **solid line** is drawn when the expression is **equal** to.

A **dotted line** is drawn when the expression is **not equal** to.

**Line 1**
\[2x + 3y > 12\]  
this line is dotted

**Line 2**
\[2x - y \leq 4\]  
this line is solid
EXCEPTION 2
The next step involves shading in the part of the graph which “satisfies” the inequality. We determine this by using one of two methods.

Method 1: POSITIVE OR NEGATIVE Y TERM
Examine the inequality and determine if y is positive or negative.

If the y term is positive, the inequality will be read as it is.
If the y term is negative, the inequality sign will be flipped.

For example,

**Line 1** \(2x + 3y > 12\) Since y is positive, y is greater than the right side.
**Line 2** \(2x - y \leq 4\) Since y is negative, the sign will flip, y is greater than the right side.

What the symbols mean:
- If y is greater than the right side, shade above the line
- If y is less than the right side, shade below the line

*Since both lines were greater than, both inequalities were shaded above the line.*

Method 2: TEST POINT METHOD
The best point to use is the origin (0, 0) provided one of the lines does not pass through this point.

**Line 1** \(2x + 3y > 12\)
Using (0, 0) \[2(0) + 3(0) > 12\]
\[0 > 12\] False (as 0 is NOT greater than 12)
∴ The side of the line containing (0,0) is NOT shaded

**Line 2** \(2x - y \leq 4\)
Using (0, 0) \[2(0) - (0) \leq 4\]
\[0 \leq 4\] True (as 0 is less than 4)
∴ The side of the line with (0,0) is shaded
Once you determine which side of each line to shade, continue shading the entire section.

The region with the double shading will be the solution (see graph below for the final solution).
Inequality systems may consist of more than two inequalities.

**Example**

The following system has five inequalities

\[ y \geq 2 \quad y \leq 8 \quad x \geq 1 \quad x \leq 7 \quad x + y \leq 10 \]

Note:

Vertical lines contain \( y \) only

Horizontal lines contain \( x \) only

Diagonal lines contain \( x \) and \( y \)

Because the rectangle is only in the first quadrant, the graph may be presented with only the first quadrant shown.
Continuing the example problem and solution from the previous page...

Find the maximum and minimum value for the expression \( C = 2x + 3y \) over the region determined by:

\[
\begin{align*}
    y & \geq 2 \\
    y & \leq 8 \\
    x & \geq 1 \\
    x & \leq 7 \\
    x + y & \leq 10
\end{align*}
\]

Step 1 Graph the inequality as in the original problem (previous page)

Step 2 Determine the vertices “corner points” of the shaded region

Reading the points from the graph:

(1, 2), (1, 8), (2, 8), (7, 3), (7, 2)

Step 3 Evaluating the points

\[
\begin{align*}
    C &= 2x + 3y \\
    2(1) + 3(2) &= 8 & \leftarrow \text{Min} \\
    2(1) + 3(8) &= 26 \\
    2(2) + 3(8) &= 28 & \leftarrow \text{Max} \\
    2(7) + 3(3) &= 23 \\
    2(7) + 3(2) &= 20
\end{align*}
\]

Answer: maximum value of 28 at (2, 8)
minimum value of 8 at (1, 2)
Problem 1

Indicate the shaded region determined by the following inequalities.

\[ y \geq 1 \]

\[ y \leq 12 \]

\[ x \geq 3 \]

\[ x \leq 15 \]

\[ 2x + 3y \leq 24 \]

Solution: Problem 1

The “corners” of the shaded regions are called vertices. (singular – vertex). The coordinates of the vertices are used in a method of problem solving called Linear Programming.
Solve systems of linear inequalities in two variables, graphically, using technology when appropriate.

This problem solving technique was developed during World War II as a way to maximize production of materials and supplies needed by the Allies. Compared to other math topic such as Geometry, Algebra, Calculus, Statistics and Trigonometry, Linear Programming is very very new.

Linear Programming can be done on computers but the term “Programming” does not have the same sense as Computer Programming. Linear Programming means a method of solving complicated problems by following a series of directions while graphing lines.

Example & Solution
Consider the irregular region shown at the right.

The vertices (corner points) are shown for the region.

A typical problem might be to evaluate the vertices for a particular relationship.

In this case the problem is to evaluate the expression \( C = 3x + 2y \) over the indicated region to find the maximum and minimum value for the expression.

To solve this problem only the vertices need to be evaluated. Since there are five vertices the expression must be evaluated with each of the five vertices and the maximum and minimum values noted.

\[ C = 3x + 2y \]

\[ 3(5) + 2(0) = 15 \leftarrow \text{Min} \]
\[ 3(3) + 2(5) = 19 \]
\[ 3(9) + 2(14) = 55 \]
\[ 3(15) + 2(8) = 61 \leftarrow \text{Max} \]
\[ 3(11) + 2(0) = 33 \]

Solution: maximum value of 61 at (15, 8)
minimum value of 15 at (5, 0)
Problem 2

Find the maximum and minimum value for the expression $T = 2y - x$ over the indicated region.

Problem 3

Find the maximum value for the expression $P = 5x + 6y$ over the indicated region.
Solutions: Problem 2

(did you notice that in the expression the “y” coordinate came first?)

\[ T = 2y - x \]

\[ 2(1) - (5) = -3 \]
\[ 2(6) - (2) = 10 \]
\[ 2(15) - (10) = 20 \leftarrow \text{Max} \]
\[ 2(8) - (16) = 0 \]
\[ 2(2) - (13) = -9 \leftarrow \text{Min} \]

Answer: maximum value of 20 at (10, 15)
minimum value of -9 at (13, 2)

Problem 3

Find the maximum value for the expression \( P = 5x + 6y \) over the indicated region.

Since the problem only asks for the maximum value there is no need to evaluate the origin since that value would be zero

\[ P = 5x + 6y \]
\[ 5(0) + 6(130) = 780 \]
\[ 5(70) + 6(90) = 890 \leftarrow \text{Max} \]
\[ 5(140) + 6(0) = 700 \]

Answer: maximum value of 890 at (70, 90)
Problem 4

Find the maximum and minimum value for the expression $C = 3x + 2y$ over the region determined by:

- $y \geq 3$
- $y \leq 15$
- $x \geq 2$
- $x \leq 9$
- $2x + 2y \leq 26$

Hint: This graph is the same one you did on page 18. Copy it here being careful when graphing the vertices. After finding the vertices evaluate the expression and determine your answer.

Problem 5

Find the maximum and minimum value for the expression $C = 2y + x$ over the region determined by:

- $y \geq 0$
- $y \leq 6$
- $x \geq 2$
- $x \leq 10$
- $y \leq -x + 12$
Solutions:

Problem 5

Vertices
(3, 1), (3, 12), (6, 12), (15, 6), (15, 1)

\[ C = 3x + 2y \]

3(3) + 2(1) = 11 \[\text{Min}\]
3(3) + 2(12) = 33
3(6) + 2(12) = 42
3(15) + 2(6) = 57 \[\text{Max}\]
3(15) + 2(1) = 47

Answer: maximum value of 57 at (15, 6)
minimum value of 11 at (3, 1)

Problem 5

Vertices
(2, 0), (2, 6), (6, 6), (10, 2), (10, 0)

\[ C = 2y + x \] (note: y is first)

2(0) + (2) = 2 \[\text{Min}\]
2(6) + (2) = 14
2(6) + (6) = 18 \[\text{Max}\]
2(2) + (10) = 14
2(0) + (10) = 10

Answer: maximum value of 18 at (6, 6)
minimum value of 2 at (2, 0)
Start by entering in your equation or inequality. This example is using the equation \( y = -2x + 8 \)

**Finding the X-intercept:**

2nd TRACE
2: zero

Find your cursor on the screen (move the LEFT and RIGHT arrow keys until it comes into view). If there are multiple equations, ensure your cursor is on the correct line (the one that is intersecting with the x-axis). Note: You can use your UP or DOWN arrow keys to move from line to line.

When the calculator asks “Left Bound?”, it is asking you to place the cursor to the “left” of the x-intercept. Once you have moved your cursor to a spot you are satisfied with, hit ENTER.

Notice how the arrow has appeared? This indicates your first boundary.

When the calculator asks “Right Bound?”, it is asking you to place the cursor to the “right” of the x-intercept. Once you have moved your cursor to a spot you are satisfied with, hit ENTER.

Notice how another arrow appeared? This indicates to the calculator the two invisible vertical boundary lines on either side of the x-intercept. Lastly, the calculator asks “Guess?”, and it is asking you to place the cursor as close to the x-intercept as possible. Once you have moved your cursor to a spot you are satisfied with, hit ENTER.

The final answer should appear now. It is the coordinate point of the x-intercept. The calculator reads Zero x=4 y=0 in this example. Therefore, the x-intercept is (4, 0).
Apply linear programming to find optimal solutions to decision-making problems

Seven Step Method

Step 1: Define the variables.

Step 2: Construct a table with the given information from the problem.

Step 3: Write the Inequalities for Quadrant 1.

Step 4: Graph the Inequalities using the intercepts method (include shading).

Step 5: Find the coordinates of the point(s) of intersections of the graphed lines (including the interceptions with the x and y axis).

Step 6: Evaluate the vertices for the relation that is to be maximized or minimized (using the profit or production equation given in the question)

Step 7: Write a statement which solves the problem.

Problem 6

A manufacturer makes long-sleeve and short-sleeve t-shirts.

Each long-sleeve shirt requires 4 minutes on the cutting machine and 3 minutes on the stitching machine.

Each short-sleeve shirt requires 3 minutes on the cutting machine and 1 minute on the stitching machine.

The cutting machine is available only 2 hours each day, and the stitching machine is available only 1 hour each day.

If the profit on a short-sleeve shirt is $6.00 and the profit on a long-sleeve shirt is $11.00, how many of each should be produced each day to maximize the profit?
**Problem 6 Solution:**

Step 1: Define the variables.

Let $L$ = the number of long-sleeve shirts produced
Let $S$ = the number of short-sleeve shirts produced

Step 2: Table of Given Information

<table>
<thead>
<tr>
<th>Type</th>
<th>Cutting (min)</th>
<th>Stitching (min)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>4</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>$S$</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>120</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

Step 3: Quadrant I inequalities

$4L + 3S \leq 120$

$3L + 1S \leq 60$

Note: you can use less time, but not more

Step 4: Graph the Inequalities.

$4L + 3S = 120$

$3L + 1S = 60$

Step 5: Find the coordinates of the corner point(s) of the shaded solution area:

- intersections of the graphed lines ($2^{nd}$ TRACE 5: Intersection)
- y-intercept ($2^{nd}$ TRACE 1: value)
- x-intercept ($2^{nd}$ TRACE 2: zeros)

Step 6: Evaluate the vertices.

Profit = $11L + 6S$

$11(0) + 6(40) = 240$

$11(12) + 6(24) = 276 \leftarrow$ Max

$11(20) + 6(0) = 220$

Step 7: Solution statement

Maximum profit of $276 by producing 12 long-sleeve t-shirts and 24 short-sleeve t-shirts
The Alford Computer Company manufactures two types of computers – Standard and Deluxe. In a day, there are 30 hours of electronic manufacturing time and 70 hours of assembly time available. Standard requires 1.5 hours of electronic manufacturing time and 1 hour of assembly time. Deluxe requires 30 minutes of electronic manufacturing time and 2 hours of assembly time. The profit on Standard is $500 and the profit on Deluxe is $360. How many of each model should be produced to achieve a maximum profit? What is the maximum profit?

Use the template to solve this problem.

Let \( x = \) 

Let \( y = \)

Profit =

Maximum profit of
Problem 8

A small manufacturing company makes chairs and footstools. Each chair requires 7 feet of lumber, each foot stool 3 feet. A chair requires 8 hours to make, while a foot stool takes 2 hours. The profit on a chair is $30 and $10 on a footstool. If 420 feet of lumber and 400 hours of labour are available, how many of each should be produced to obtain a maximum profit? What is the maximum profit?

Use the template to solve this problem.

Let $x =$

Let $y =$

Profit =

Maximum profit of
**Problem 9**

Josie is on a diet. Daily, she needs three dietary supplements, A, B and C as follows: at least 16 units of A, 6 units of B, and 20 units of C. These can be found in either of two marketed products Squabb I which contains 2 units of A, 1 unit of B and 5 units of C and Squabb II which contains 4 units of A, 1 unit of B and 2 units of C. The Squabb I pill cost $1.50 and the Squabb II pill costs $2.00. How many of each pill should Josie buy to satisfy her dietary needs and at the same time minimize costs?

How is this problem different than the previous examples? This problem is different in two ways.

First, there are three limiting factors \(\rightarrow\) supplements A, B and C.

Second, the problem ask you to minimize costs not maximize profits.

Let \(x=\quad\)

Let \(y=\quad\)

Profit = Minimize costs of
**Problem 7**

Let

\[ x = \text{# of Standard Computers produced} \]
\[ y = \text{# of Deluxe Computers produced} \]

1.5\(x\) + 0.5\(y\) \(\leq\) 30
1\(x\) + 2\(y\) \(\leq\) 70

Graphing Calculator entry:

\[
y \leq -\frac{1.5x}{0.5} + \frac{30}{0.5}
\]
\[
y \leq -\frac{x}{2} + \frac{70}{2}
\]

Profit = $500 S + $360 D

\[
\begin{align*}
$500 \times 0 +$360 \times 35 &= $10 500 \\
$500 \times 10 +$360 \times 30 &= $14 000 \quad \text{Max} \\
$500 \times 20 +$360 \times 0 &= $10 000
\end{align*}
\]

Maximum profit of $14 000 obtained by producing 10 Standard and 30 Deluxe Computers.
Problem 8

Let
\( x = \# \text{ of chairs produced} \)
\( y = \# \text{ of footstools produced} \)

\[
\begin{align*}
7x + 3y &\leq 420 \\
8x + 2y &\leq 400
\end{align*}
\]

Graphing Calculator entry:

\[
\begin{align*}
y &\leq -\frac{7x}{3} + \frac{420}{3} \\
y &\leq -\frac{8x}{2} + \frac{400}{2}
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Lumber (feet)</th>
<th>Time (hours)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chairs</td>
<td>7</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Footstools</td>
<td>3</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

Profit = $30 \ C + $10 \ F

\[
\begin{align*}
$30 (0) + $10 (140) &= $1\ 400 \\
$30 (36) + $10 (56) &= $1\ 640 \\
$30 (50) + $10 (0) &= $1\ 500
\end{align*}
\]

Maximum profit of $1\ 640 by producing 36 chairs and 56 footstools.
**Problem 9**

Let

\[ x = \text{# of Squibb I pills} \]

\[ y = \text{# of Squibb II pills} \]

**Three inequalities**

\[ 2x + 4y \geq 16 \]

\[ x + y \geq 6 \quad \text{(greater than or equal because you can take more but not less)} \]

\[ 5x + 2y \geq 20 \]

- **Graphing Calculator entry:**

  \[ y \geq -\frac{2x}{4} + \frac{16}{4} \]

  \[ y \geq -x + 6 \]

  \[ y \geq -\frac{5x}{2} + \frac{20}{2} \]

**Note: this time we are evaluating for minimum values**

\[ C = 1.5X + 2Y \]

- \$1.5(0) + 2(10) = \$20
- \$1.5(2) + 2(3) = \$9 \quad \text{Min}
- \$1.5(4) + 2(2) = \$10
- \$1.5(8) + 2(0) = \$16

Minimum cost of \$9 by taking 2 Squibb I pills and 3 Squibb 2 pills

Note: Not all intersections are integer values
1. Bob builds tool sheds. He uses 10 sheets of dry wall and 15 studs for a small shed and 15 sheets of dry wall and 45 studs for a large shed. He has available 60 sheets of dry wall and 135 studs. If Bob makes $390 profit on a small shed and $520 on a large shed, how many of each type of building should Bob build to maximize his profit?

2. A company makes a product in two different factories. At factory X it takes 30 hours to produce the product and at factory Y it takes 20 hours. The costs of producing these items are $50 at factory X and $60 at factory Y. The company’s labor force can provide 6000 hours of labor each week and resources are $12,000 each week. How should the company allocate its labor and resources to maximize the number of products produced? How many hours would be used at factory X? How many dollars would be spent at factory Y?

3. An agriculture company has 80 tons of type I fertilizer and 120 tons of type II fertilizer. The company mixes these fertilizers into two products. Product X requires 2 parts of type I and 1 part of type II fertilizers. Product Y requires 1 part of type I and 3 parts of type II fertilizers. If each product sells for $2000, what is the maximum revenue and how many of each product should be made and sold to maximize revenue?

4. Two men, John and Bill, make math supplies for schools. In one hour, John can make 10 protractors, 5 compasses, and 5 metre sticks. Bill can make 5 protractors, 5 compasses, and 20 metre sticks in the same time. They have an order for 40 protractors, 80 compasses, and 110 metre sticks. John earns $8.50 per hour, and Bill earns $12.50 per hour. How long should each work to minimize the cost of labour?

5. Ivan and his daughter service lawn equipment such as lawn mowers and weed trimmers. Ivan can service a lawn mower in 15 minutes and a weed trimmer in 12 minutes. His daughter can service a lawn mower in 10 minutes and a weed trimmer in 20 minutes. Ivan is paid $12.00/hour and his daughter is paid $8.00/hour. If they have an order for 50 lawn mowers and 40 weed trimmers, how long should each work to minimize the cost of labour?
Practice Test

Show all work for full credit.

1. (4 pts) Solve the following inequalities. Show your answer on the number line and by using interval notation.

a) \[ 5x + 3 \leq 4 + 4x \]

b) \[ 4x + 3 < 11 + 6x \]

2. (4 pts) Solve the following system of inequalities. Indicate your answer.

\[ y < 3x - 4 \]

\[ y \leq -3x + 2 \]
3. (5 pts) Solve the following system by graphing.

\[3x + y > 9\]

\[x + 2y > 8\]

4. (4 pts) Write the defining system of Inequalities for the solution region that is shaded in the diagram.

Line a _____________________

Line b _____________________
5. (7 pts) Using corner points, find the maximum and minimum value for the expression \( T = y + 3x \) over the indicated region.

\[
\begin{align*}
y &\geq 2 \\
y &\leq 10 \\
y &\leq -2x + 14 \\
x &\geq 1 \\
x &\leq 5
\end{align*}
\]

6. (5 pts) Find the maximum and minimum values for the expression \( C = 3x - 2y \) over the indicated region.
7. (7 pts) A furniture manufacturer makes sofas and chairs. The steps in the production process are carpentry and upholstering. Making a sofa takes 3 hours for carpentry and 6 hours for upholstering. Making a chair takes 6 hours for carpentry, and 2 hours for upholstering. In one day, 96 hours are available for carpentry, and 72 hours for upholstering. The profit per chair is $80 and the profit per sofa is $70. How many sofas and chairs should be made in a given day to maximize profit? (Must show ALL seven steps.)
8. (7 pts) A company manufactures two types of hats. The first type of hat requires 6 minutes of cutting time and 1 minute of sewing time. The second type of hat requires 2 minutes of cutting time and 4 minutes of sewing time. In each 2-hour time slot, the company has a maximum of 75 minutes of cutting time and 40 minutes of sewing time for the production of hats. The company makes a profit of $8 on each of the first type of hat and $5 on each of the second type of hat. Determine how many of each type of hat the company should produce in each 2-hour time slot to maximize its profits. Is the result reasonable? If not, explain how you would choose a reasonable number of hats of each to produce. (Must show ALL seven steps.)
9. (7 pts) The triple A Toy manufacturing company, with two production plants, produces only one toy, a small plastic doll. It costs $5 to produce one doll at the plant in Winkler and $6 to produce one doll at the plant in Altona. The company has 900 person hours per week of labour time available, and has set a maximum production cost of $1500. It takes 4 hours to produce one item in Winkler and 3 hours to produce one doll in Altona. Find the optimum (maximum) number of dolls that can be produced in 1 week and the number of hours of production in Altona and Winkler.

Hours in Altona  ______________

Hours in Winkler ______________

Maximum Production _____________