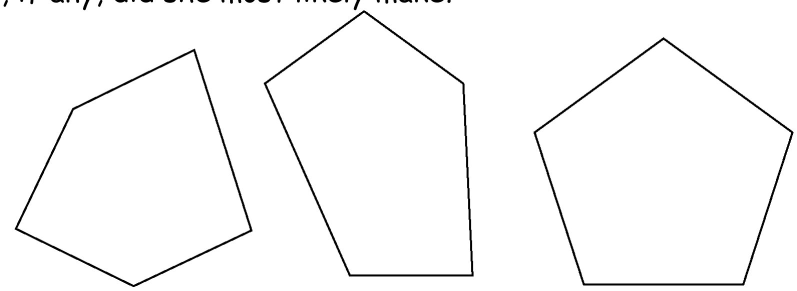
Lesson 1 - Conjectures

____ 1. Which conjecture, if any, could you make about the product of an odd integer and an even integer?

a. The product will be an even integer.
b. The product will be an odd integer.
c. The product will be negative.
d. It is not possible to make a conjecture.

____ 2. Eileen studied the sum of the angles in pentagons and made a conjecture. Which conjecture, if any, did she most likely make?



a. The sum of the angles in a pentagon is always 180°.
b. The sum of the angles in a pentagon is always 360°.
c. The sum of the angles in a pentagon is always 540°.
d. It is not possible to make a conjecture.

____ 3. Which conjecture, if any, could you make about the sum of two even integers and one odd integer?

a. The sum will be an odd integer.
b. The sum will be an even integer.
b. The sum will be negative.
d. It is not possible to make a conjecture.

____ 4. Lila created the following table.

Multiples of 5	15	75	150
Sum of the Digits	6	12	6

Based on this evidence, which conjecture might Lila make? Is the conjecture valid?

a. The sum of the digits of a multiple of 3, is a multiple of 6; no, this conjecture is not valid.
b. The sum of the digits of a multiple of 3 is a multiple of 6; yes, this conjecture is valid.
c. The sum of the digits of a multiple of 5 is a multiple of 6; yes, this conjecture is valid.
d. The sum of the digits of a multiple of 5, is a multiple of 6; no, this conjecture is not valid.

____ 5. Justin gathered the following evidence.

17(22) = **374**   14(22) = **308**   36(22) = **742**   18(22) = **396**

Which conjecture, if any, is Justin most likely to make from this evidence?

a. When you multiply a two-digit number by 22, the last and first digits of the product are the digits of the original number.
b. When you multiply a two-digit number by 22, the first and last digits of the product are the digits of the original number.
c. When you multiply a two-digit number by 22, the first and last digits of the product form a number that is twice the original number.
d. None of the above conjectures can be made from this evidence.

____ 6. Which conjecture, if any, could you make about the product of two odd integers?

a. The product will be an even integer.
b. The product will be an odd integer.
c. The product will be negative.
d. It is not possible to make a conjecture.

____ 7. Jason created the following table to show a pattern.

Multiples of 27	54	81	108	135	162
Sum of the Digits	9	9	9	9	9

Which conjecture could Jason make, based solely on this evidence? Choose the best answer.

a. The sum of the digits of a multiple of 27 is equal to 9.
b. The sum of the digits of a multiple of 27 is an odd integer.
c. The sum of the digits of a multiple of 27 is divisible by 9.
d. Jason could make any of the above conjectures, based on this evidence.

____ 8. Emma works part-time at a bakery shop in Saskatoon. Today, the baker made 20 apple pies, 20 cherry pies, and 20 bumbleberry pies. Which conjecture is Emma most likely to make from this evidence?

a. People are more likely to buy bumbleberry pie than any other pie.
b. People are more likely to buy apple pie than any other pie.
c. Each type of pie will sell equally as well as the others.
d. People are more likely to buy cherry pie than any other pie.

____ 9. Gary works at a bicycle store in Vancouver. For the start of spring, the manager of the store has ordered 50 mountain bikes and 10 racing bikes.

Which conjecture is Gary most likely to make from this evidence?

a. Either type of bike will sell equally well.
b. Racing bikes will likely sell better than mountain bikes.
c. It will rain all summer and no one will ride bicycles.
d. Mountain bikes will likely sell better than racing bikes.

____ 10. Jessica noticed a pattern when dividing these numbers by 4: 5^3, 9^3, 13^3.

Determine the pattern and make a conjecture.

a. When the cube of an odd number that is 1 more than a multiple of 4 is divided by 4, the decimal part of the result will be .75.
b. When the cube of an odd number that is 1 less than a multiple of 4 is divided by 4, the decimal part of the result will be .75.
c. When the cube of an odd number that is 1 more than a multiple of 4 is divided by 4, the decimal part of the result will be .25.
d. When the cube of an odd number that is 1 less than a multiple of 4 is divided by 4, the decimal part of the result will be .25.

11. While driving along the road one morning, Jenny noticed that all the cows in a field were standing up, with their heads pointing northward. In the afternoon, it started to snow. Jenny made the conjecture that when cows stand and face northward, it will likely snow. Is Jenny’s conjecture reasonable? Briefly justify your decision.

12. What conjecture could you make about the product of two odd integers and one even integer?

13. Perry works at a bakery shop in Regina. He goes to the farmer's market and buys enough fruit to bake 20 saskatoon berry pies and 10 rhubarb pies. Which conjecture has Perry most likely made?

14. Kathryn texts this message to her friend Jamie: **1 2** $\frac{1}{2}$ **dinner L8R**?

Jamie responds: **GR8! CU FTR WRK**. Make a conjecture about what was said.

Lesson 2 - Valid Conjectures

1. Kerry created the following tables to show patterns.

Multiples of 3	12	15	18	21
Sum of the Digits	3	6	9	3

In each case, the sum of the digits of a multiple of 3 is also a multiple of 3.

Multiples of 3 • 3 = 9	18	27	36	45
Sum of the Digits	9	9	9	9

In each case, the sum of the digits of a multiple of 3 • 3, or 9, is also a multiple of 9.

Based on this evidence, which conjecture might Kerry make? Is the conjecture valid?

a. The sum of the digits of a multiple of 2 • 3, or 6, is also a multiple of 6; yes, this conjecture is valid.
b. The sum of the digits of a multiple of 2 • 3, or 6, is also a multiple of 6; no, this conjecture is not valid.
c. The sum of the digits of a multiple of 3 • 3 • 3, or 27, is also a multiple of 27; no, this conjecture is not valid.
d. The sum of the digits of a multiple of 3 • 3 • 3, or 27, is also a multiple of 27; yes, this conjecture is valid.

2. Lila created the following table.

Multiples of 5	15	75	150
Sum of the Digits	6	12	6

Based on this evidence, which conjecture might Lila make? Is the conjecture valid?

a. The sum of the digits of a multiple of 3, is a multiple of 6; no, this conjecture is not valid.
b. The sum of the digits of a multiple of 3 is a multiple of 6; yes, this conjecture is valid.
c. The sum of the digits of a multiple of 5 is a multiple of 6; yes, this conjecture is valid.
d. The sum of the digits of a multiple of 5, is a multiple of 6; no, this conjecture is not valid.

3. Hedly gathered the following evidence. 5(22) = **110** 5(33) = **165** 5(44) = **220**

   Which conjecture might Hedly make from this evidence? Is the conjecture reasonable?

   a. When you multiply 11 by a multiple of 11, the first and last digits of the product will sum to the middle digit; yes, this conjecture is valid.
   b. When you multiply 5 by a multiple of 11, the first and last digits of the product will sum to the middle digit; no, this conjecture is not valid.
   c. When you multiply 5 by a multiple of 11, the first and last digits of the product will sum to the middle digit; yes, this conjecture is valid.
   d. When you multiply 11 by a multiple of 5, the first and last digits of the product will sum to the middle digit; no, this conjecture is valid.

4. Jimmy claims that whenever you square an even integer, the result is an even number. Is his conjecture reasonable? Briefly justify your decision.

5. Star claims that whenever you add an odd integer to the square of an odd integer, the result is an odd number. Is her conjecture reasonable? Briefly justify your decision.

6. Xavier claims that whenever you add an even integer to the square of an even integer, the result is an even number. Is his conjecture reasonable? Briefly justify your decision.

7. During the 2010 World Cup tournament, an octopus named Paul correctly predicted which teams would win several games. Make a conjecture based on this evidence. Is this conjecture reasonable

8. Edward gathered the following evidence: 4(33) = **132** 5(33) = **165** 6(33) = **198**

From this evidence, Edward made the following conjecture.

When you multiply a one-digit number by 33, the first and last digits of the product form a number that is three times the original number.

Is his conjecture reasonable? Briefly justify your decision.

9. Katy gathered the following evidence: 17(22) = **374** 36(22) = **742** 18(22) = **396**

From this evidence, Katy made the following conjecture.

When you multiply a two-digit number by 22, the first and last digits of the product form a number that is twice the original number.

Is her conjecture reasonable? Briefly justify your decision.

10. Jason created the following table to show a pattern.

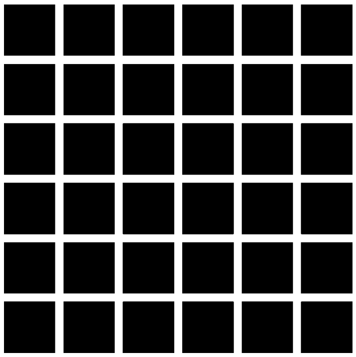
Multiples of 27	54	81	108	135	162
Sum of the Digits	9	9	9	9	9

Based on this evidence, Jason made the following conjecture:

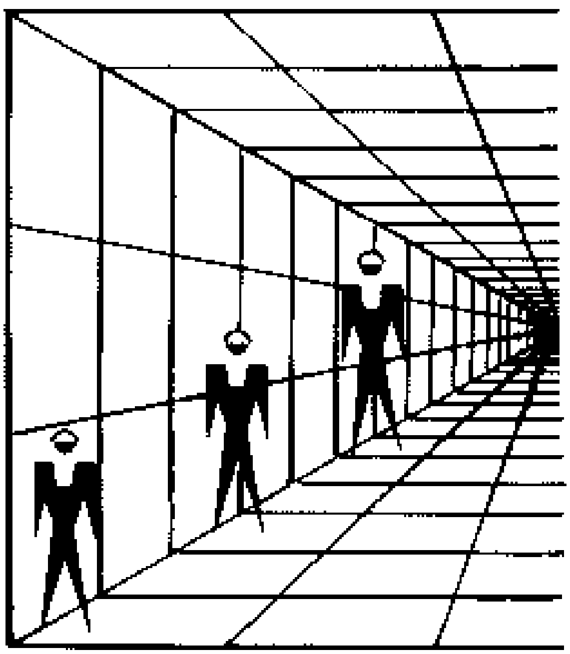
The sum of the digits of a multiple of 27 is equal to 9.

Try more examples. Is this conjecture reasonable? Briefly justify your decision.

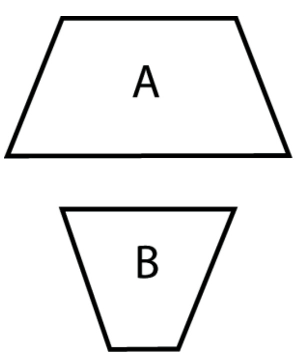
11. Make a conjecture about the colour of the spaces between the black squares. Check the validity of your conjecture.



12. Make a conjecture about the relative size of the three figures. Check the validity of your conjecture.



13. Which figure has the longer top side, A or B? Make a conjecture and check the validity of your conjecture.



Lesson 3 - Finding Counterexamples to Conjectures

____ 1. Bill made the following conjecture:

When you add a multiple of 6 and a multiple of 9, the sum will be a multiple of 6.

Is the following equation a counterexample to this conjecture? **12 + 27 = 39** Explain.

a. Yes, it is a counterexample, because 39 is not a multiple of 6.
b. Yes, it is a counterexample, because 39 is a multiple of 3.
c. No, it is not a counterexample, because 39 is a multiple of 3.
d. No, it is not a counterexample, because 39 is not a multiple of 9.

____ 2. Jackie made the following conjecture.

The square of a number is always greater than the number.

Which choice, if either, is a counterexample to this conjecture?

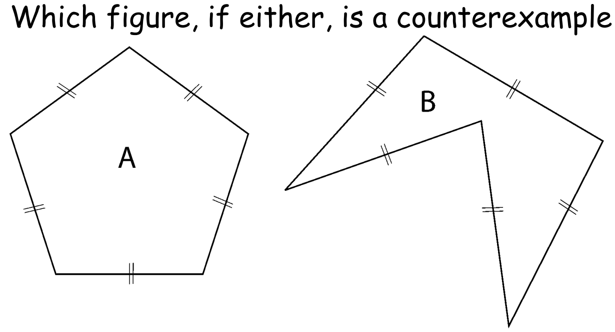
1. $0.5^2 = 0.25$ **2.** $(-5)^2 = 25$

a. Choice 1 and Choice 2 b. Choice 2 only
c. Neither Choice 1 nor Choice 2 d. Choice 1 only

____ 3. Rosie made the following conjecture:

All polygons with five equal sides are regular pentagons.

Which figure, if either, is a counterexample to this conjecture?



a. Figure B only b. Figure A only
c. Neither Figure A nor Figure B d. Figure A and Figure B

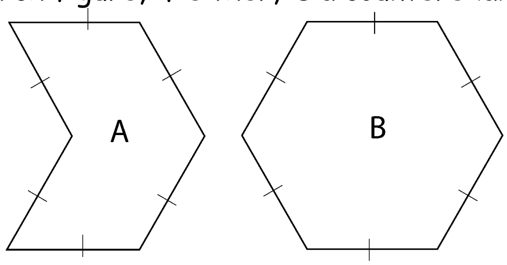
4. Anna's little sister, Joyce, made the following conjecture:

All girls' names either begin or end with a vowel.

Use a counterexample to show Joyce her conjecture is not valid.

____ 5. Sasha made the following conjecture: **All polygons with six equal sides are regular hexagons.**

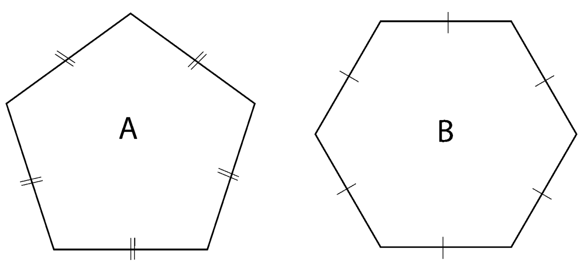
Which figure, if either, is a counterexample to this conjecture? Explain.



a. Figure A is a counterexample, because all six sides are equal and it is a regular hexagon.
b. Figure B is a counterexample, because all six sides are equal and it is a regular hexagon.
c. Figure B is a counterexample, because all six sides are equal and it is not a regular hexagon.
d. Figure A is a counterexample, because all six sides are equal and it is not a regular hexagon.

____ 6. Tashi made the following conjecture: **All polygons with equal sides are regular.**

Which figure, if either, is a counterexample to this conjecture?



a. Figure A and Figure B
b. Figure B only
b. Neither Figure A nor Figure B
d. Figure A only

____ 7. Siddartha made the following conjecture.

When you divide two whole numbers, the quotient will be greater than the divisor and less than the dividend.

Which choice, if either, is a counterexample to this conjecture?

1. $\frac{4}{8} = 0.5$

2. $\frac{12}{4} = 3$

a. Choice 2 only
b. Choice 1 and Choice 2

c. Choice 1 only   d. Neither Choice 1 nor Choice 2

____ 8. Randolph made the following conjecture.

The sum of a multiple of 4 and a multiple of 8 must be a multiple of 2.

Which choice, if either, is a counterexample to this conjecture?

1. $4 + 8 = 12$ **2.** $8 + 8 = 16$

a. Choice 2 only   b. Choice 1 and Choice 2
c. Choice 1 only   d. Neither Choice 1 nor Choice 2

____ 9. Pearl made the following conjecture.

The sum of a multiple of 4 and a multiple of 8 must be a multiple of 8.

Is the following equation a counterexample to this conjecture? **$24 + 24 = 48$** Explain.

a. No, it is not a counterexample, because 48 is not a multiple of 8.
b. No, it is not a counterexample, because 48 is a multiple of 8.
c. Yes, it is a counterexample, because 48 is not a multiple of 8.
d. Yes, it is a counterexample, because 48 is a multiple of 8

____ 10. Henry made the following conjecture:

The square of a number is always greater than the number.

Is the following equation a counterexample to this conjecture? **$0.4^2 = 0.16$** Explain.

a. Yes, it is a counterexample, because 0.4 is less than 0.16.
b. No, it is not a counterexample, because 0.16 is less than 0.4.
c. No, it is not a counterexample, because 0.4 is less than 0.16.
d. Yes, it is a counterexample, because 0.16 is less than 0.4.

____ 11. Ginerva made the following conjecture:

The square of a number is always greater than the number.

Is the following equation a counterexample to this conjecture? **$5^2 = 25$** Explain.

a. No, it is not a counterexample, because 25 is greater than 5.
b. No, it is not a counterexample, because 25 is less than 5.
c. Yes, it is a counterexample, because 25 is greater than 5.

d. Yes, it is a counterexample, because 5 is less than 25.

____ 12. Attila made the following conjecture:

The difference between two numbers always lies between the two numbers.

Is the following equation a counterexample to this conjecture? **6 – 2 = 4** Explain.

a. No, it is not a counterexample, because 4 lies between 2 and 6.
b. Yes, it is a counterexample, because 4 does not lie between 2 and 6.
c. Yes, it is a counterexample, because 4 lies between 2 and 6.
d. No, it is not a counterexample, because 4 does not lie between 2 and 6.

____ 13. Rachelle made the following conjecture:

When you add a multiple of 6 and a multiple of 9, the sum will be a multiple of 3.

Is the following equation a counterexample to this conjecture? **21 + 27 = 48** Explain.

a. Yes, it is a counterexample, because 48 is not a multiple of 3.
b. No, it is not a counterexample, because 21 is not a multiple of 9.
c. No, it is not a counterexample, because 21 is not a multiple of 6.
d. Yes, it is a counterexample, because 48 is a multiple of 3.

14. Elinor made the following conjecture: **All of Canada's prime ministers have been men.**

Do you agree or disagree? Justify your decision with a counterexample if possible.

15. Cheyenne told her little brother, Joseph, that horses, cats, and dogs are all mammals.
As a result, Joseph made the following conjecture: **All animals with four legs are mammals.**

Use a counterexample to show Joseph that his conjecture is not valid.

16. Austin told his little sister, Celina, that horses, cats, and dogs are all mammals.
As a result, Celina made the following conjecture: **All mammals have four legs.**

Use a counterexample to show Celina her conjecture is not valid.

18. Tyler made the following conjecture:

A polygon with more than two right angles must be a rectangle.

Do you agree or disagree? Briefly justify your decision with a counterexample if possible.

Lesson 4 - Proving Conjectures: Deductive Reasoning

____ 1. All cats are mammals. All mammals are warm-blooded. Tashi is a cat. What can be deduced about Tashi?

1. Tashi is warm-blooded. **2.** Tashi is a mammal.

a. Choice 1 and Choice 2
b. Neither Choice nor Choice 2
c. Choice 1 only
d. Choice 2 only

____ 2. All alligators are reptiles. All reptiles are covered with scales. Tashi is a cat. What can be deduced about Tashi?

1. Tashi has scales. **2.** Tashi is a reptile.

a. Choice 1 and Choice 2
b. Neither Choice 1 nor Choice 2
c. Choice 1 only
d. Choice 2 only

____ 3. Isabelle is a manicurist. Everyone whose nails are done by Isabelle gets a good manicure. Ginerva's nails were done by Isabelle. What can be deduced about Ginerva?

1. Ginerva has a good manicure. **2.** Ginerva is a manicurist.

a. Choice 1 and Choice 2
b. Neither Choice nor Choice 2
c. Choice 1 only
d. Choice 2 only

____ 4. Hali is a fitness instructor. People who take Hali's exercise class regularly soon become very fit. Regular exercise makes people feel happy. Joshua takes Hali's exercise class regularly. What can be deduced about Joshua?

1. Joshua is very fit. **2.** Joshua feels happy.

a. Choice 1 and Choice 2
b. Neither Choice nor Choice 2
d. Choice 1 only
d. Choice 2 only

____ 5. Which of the following choices, if any, uses inductive reasoning to show that the sum of three even integers is even?

a. $2x + 2y + 2z = 2(x + y + z)$
b. 2 + 4 + 6 = 12 and 4 + 6 + 8 = 18
c. $x + y + z = 2(x + y + z)$
d. None of the above choices

____ 6. Which of the following choices, if any, uses deductive reasoning to show that the sum of two odd integers is even?

a. $3 + 5 = 8$ and $7 + 5 = 12$
b. $(2x + 1) + (2y + 1) = 2(x + y + 1)$
c. $2x + 2y + 1 = 2(x + y) + 1$
d. None of the above choices

____ 7. Which of the following choices, if any, uses inductive reasoning to show that an odd number and an odd number sum to an even number?

a. $2x + 2y = 2(x + y)$
b. $2x + 2y + 1 = 2(x + y) + 1$
c. $6 + 6 = 12$ and $4 + 6 = 10$
d. None of the above choices

8. All camels are mammals. All mammals have lungs to breathe air. Humphrey is a camel. What can be deduced about Humphrey?

9. What can you deduce about the sum of two even numbers and an odd number? Briefly explain your answer using deductive reasoning.

10. Prove, using deductive reasoning, that the product of an even integer and an even integer is always even.

11. Sir Arthur Conan Doyle wrote the mystery story "Silver Blaze". In this story, Sherlock Holmes investigates the theft of a famous race horse.

Holmes learns the following facts:
- The family dog did not bark when the crime happened.
- The crime happened at night.
- The family dog always barks at strangers at night.

What deduction could Holmes make about who committed the crime? Explain briefly.

12. Try the following number trick with different numbers. Make a conjecture about the trick.

    - Choose a number.
    - Multiply by 3.
    - Add 5.
    - Multiply by 2.
    - Subtract 10.
    - Divide by 6.

13. Complete the conclusion for the following deductive argument:

    If an even integer is not divisible by 4, then half the number is an odd number.
    14 is not divisible by 4, therefore, ...

14. Diego wrote the following proof. What conjecture could he be trying to prove?

    Let n be the any integer.

    $n^2 - (n - 1)(n + 1) = n^2 - (n^2 + n - n - 1)$
    $n^2 - (n - 1)(n + 1) = n^2 - n^2 + 1$
    $n^2 - (n - 1)(n + 1) = 1$

15. Try the following calculator trick with different numbers. Make a conjecture about the trick.

    - Start with your age.
    - Multiply it by 3.
    - Multiply it by 7.
    - Multiply it by 37.
    - Multiply it by 13.

16. Does the following statement demonstrate inductive reasoning or deductive reasoning?

    For the past three years, a bush has produced roses. Therefore, the bush will produce roses this year.

17. Does the following statement demonstrate inductive reasoning or deductive reasoning?

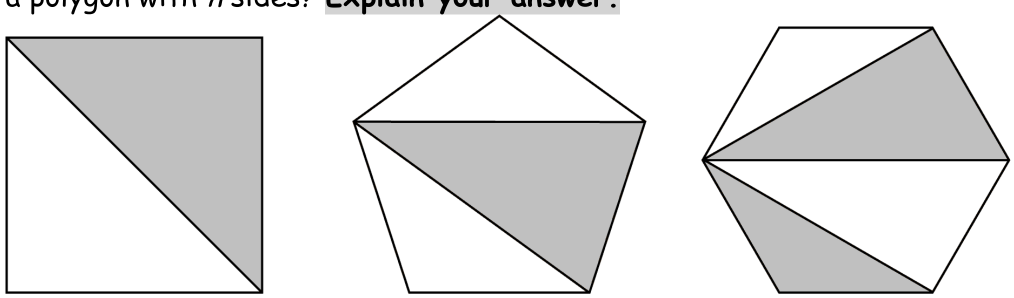
Every Monday afternoon at 6:00 p.m., the news is broadcast on television. Today is Monday, therefore, the news will be broadcast on television.

18. Does the following statement demonstrate inductive reasoning or deductive reasoning?
For the pattern 4, 13, 22, 31, 40, the next term is 49.

19. Does the following statement demonstrate inductive reasoning or deductive reasoning?

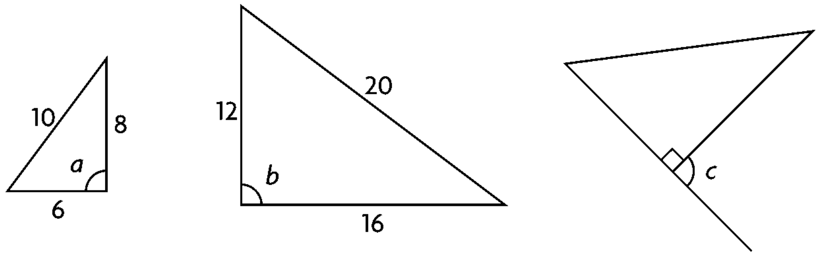
Every high school student in western Canada has to take math.
You are a high school student in western Canada, therefore you have to take math.

20. Lila drew some polygons with triangles inside them, as shown. She recorded the sum of the angles in each polygon. What could Lily deduce about the sum of the angles in a polygon with *n* sides? **Explain your answer.**



sides: 4	sides: 5	sides: 6
triangles: 2	triangles: 3	triangles: 4
angle sum: 360°	angle sum: 540°	angle sum: 720°

21. Prove that *a*, *b*, and *c* are equal.



22. Alison discovered a number trick in a book she was reading:

    Choose a number.
    Add 3.
    Multiply by 2.
    Add 4.
    Divide by 2.
    Subtract 5.

    Prove deductively that any number you choose will be the final result.

23. Star claims that whenever you add seven consecutive integers, the sum is always 5 times the median of the numbers. Is Star's conjecture true or not? Prove your answer.

24. The divisibility rule for 9 is:

    If the sum of the digits of a number is divisible by 9, then the original number is divisible by 9.

    Prove this rule is true for numbers with three digits.

Lesson 5 - Proofs That Are Not Valid

____ 1. What type of error, if any, occurs in the following deduction?

All people drive cars to work. Gavin drives to work. Therefore, Gavin drives a car.

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

____ 2. What type of error, if any, occurs in the following proof?

$$\begin{aligned} 2 &= 2 + 2 \\ 4(2) &= 4(2 + 2) \\ 4(2) + 3 &= 4(2 + 2) + 3 \\ 8 + 3 &= 16 + 3 \\ 11 &= 19 \end{aligned}$$

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

____ 3. What type of error, if any, occurs in the following proof?

$$\begin{aligned} 3 &= 3 - 1 \\ 2(3) &= 2(3 - 1) \\ 2(3) + 1 &= 2(3 - 1) + 1 \\ 6 + 1 &= 4 + 1 \\ 7 &= 5 \end{aligned}$$

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

____ 4. What type of error, if any, occurs in the following deduction?

If you combine one haystack with another haystack, you get one haystack. Therefore, 1 + 1 = 1.

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

_____ 5. What type of error, if any, occurs in the following proof?

Suppose that: $x + y = z$

Then: $(3x - 2x) + (3y - 2y) = (3z - 2z)$

Reorganize: $3x + 3y - 3z = 2x + 2y - 2z$

Using distribution: $3(x + y - z) = 2(x + y - z)$

$3 = 2$

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

_____ 6. What type of error, if any, occurs in the following proof?

SIX = 6	I know that "six" and "6" are different ways of writing the same thing.
IX = 9	In roman numerals, IX represents the number 9.
SIX has more letters than IX.	Therefore, SIX must be greater than IX.
6 > 9	

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

_____ 7. Alison created a number trick in which she always ended with the original number. When Alison tried to prove her trick, however, it did not work. What type of error occurs in the proof?

n	**Use *n* to represent any number.**
$n + 4$	**Add 4.**
$2n + 4$	**Multiply by 2.**
$2n + 8$	**Add 4.**
$n + 4$	**Divide by 2.**
$n - 1$	**Subtract 5.**

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

____ 8. What type of error, if any, occurs in the following deduction?

All students who graduate from Urban City High School with an average of 95% or more will get their photograph in the local newspaper. Jerome graduated from Urban City High School with a 96% average. Therefore, Jerome will get his photograph in the local newspaper.

a. false assumption or generalization
b. an error in reasoning
c. an error in calculation
d. There is no error in the deduction.

9. Examine the following example of deductive reasoning. Why is it faulty?

Given: All islands are surrounded by water. Whales are surrounded by water.
Deduction: Whales are islands.

10. Bradley wanted to prove something about the sum of any six consecutive natural numbers. He wrote this equation:

$x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) = 6x + 15$

What has Bradley proven?

11. Examine the following example of deductive reasoning. Why is it faulty?

Given: At 11:00 p.m. this evening, there will be a newscast on Channel 20. There is a newscast on Channel 20 starting right now.
Deduction: It is now 11:00.

12. Examine the following example of deductive reasoning. Why is it faulty?

Given: TechShop sells computers. Aaron wants to buy new headphones.
Deduction: Aaron should not shop at TechShop.

13. What type of error occurs in the following deduction? Briefly justify your answer.

All videos have large groups of dancers. The western band is recording a new video, so it must have a large group of dancers.

14. What type of error occurs in the following proof? Briefly justify your answer.

$$\begin{aligned} 7 &= 7 - 1 \\ 2(7) + 3 &= 2(7 - 1) + 3 \\ 14 + 3 &= 2(6) + 3 \\ 17 &= 12 + 3 \\ 17 &= 15 \end{aligned}$$

15. What type of error occurs in the following deduction? Briefly justify your answer.

Given: Cheryl made blueberry muffins.
Blueberry muffin recipes call for flour and milk.
Anton is baking carrot muffins.
Deduction: Anton's recipe does not call for flour and milk.

16. What type of error occurs in the following deduction? Briefly justify your answer.

People wear hats to prevent sunstroke.
Eldon is wearing a hat.
Therefore, Eldon is wearing the hat to prevent sunstroke.

17. What type of error occurs in the following deduction? Briefly justify your answer.

Let $x = y$.

$$
\begin{aligned}
x^2 &= xy \\
x^2 + x^2 &= x^2 + xy \\
2x^2 &= x^2 + xy \\
2x^2 - 2xy &= x^2 + xy - 2xy \\
2x^2 - 2xy &= x^2 - xy \\
2(x^2 - xy) &= 1(x^2 - xy) \\
2 &= 1
\end{aligned}
$$

18. What type of error occurs in the following deduction? Briefly justify your answer.

$$
\begin{aligned}
3 &= 3 \\
7(3) &= 7(2 + 1) \\
7(3) + 6 &= 7(2 + 1) + 6 \\
21 + 6 &= 14 + 7 \\
27 &= 21
\end{aligned}
$$

19. Alison created a number trick in which she always ended with the original number. When Alison tried to prove her trick, however, it did not work. In which step does the calculation error occur? What is the error?

n	**Use n to represent any number.**
$n + 4$	**Add 4.**
$2n + 4$	**Multiply by 2.**
$2n + 8$	**Add 4.**
$n + 4$	**Divide by 2.**
$n - 1$	**Subtract 5.**

20. Terry wrote the following equations. The results led him to make the conjecture that the sum of three consecutive perfect cubes is not divisible by 3.

Let n be the first integer.

$$n^3 + (n + 1)^3 + (n + 2)^3 = n^3 + (n^3 + 3n^2 + 3n + 1) + (n^3 + 6n^2 + 12n + 8)$$
$$n^2 + (n + 1)^2 + (n + 2)^2 = 3n^3 + 9n^2 + 15n + 10$$
$$n^2 + (n + 1)^2 + (n + 2)^2 = (3n^3 + 9n^2 + 15n + 9) + 1$$
$$n^2 + (n + 1)^2 + (n + 2)^2 = 3(n^3 + 3n^2 + 5n + 3) + 1$$

But in checking his work, Terry found the following counterexample, so he knew he had made an error.

1 + 8 + 27 = 36

Determine what Terry's error is. What does the proof show?

21. Six people enjoyed a meal at an East Indian restaurant. The waiter brought a bill for \$60. Each person at the table paid \$10. Later the manager realized that the bill should have been for only \$50, so she sent the waiter back to the table with \$10. The waiter could not figure out how to divide \$10 six ways, so he gave each person \$1 and kept \$4 for himself. Each of the six people paid \$9 for the meal.
$9 \cdot 6 = 54$
The waiter kept \$4.
$54 + 4 = 58$
What happened to the other two dollars?

22. Kim tried this number trick:
    - Write down the number of your birth month.
    - Multiply by 2.
    - Add the number of days in a week.
    - Multiply by 50.
    - Add the last two digits of your birth year.
    - Subtract the number of days in a year.
    - Add 15.

    Kim's result was a number in which the tens and ones digits were her birth year and the rest of the digits was her address. She tried to prove why this works, but her final expression did not make sense.

    Let n represent any address.

$2n$	Multiply by 2.
$2n + 7$	Add the number of days in a week.
$100n + 350$	Multiply by 50.
Let y represent the last two numbers of your birth year.	
$100n + 350 + y$	Add your birth year.
$100n + 350 + y - 366$	Subtract the number of days in a non-leap year.
$100n + p - 1$	Add 15.

    Determine the errors in Kim's proof, and then correct them.

23. Michale says she can prove that 4 = 0. Here is her proof.

    Let both x and y be equal to 1.

Step 1:	$x = y$
Step 2:	$x^2 = y^2$
Step 3:	$x^2 - y^2 = 0$
Step 4:	$(x - y)(x + y) = 0$
Step 5:	$\dfrac{(x-y)(x+y)}{(x-y)} = \dfrac{0}{(x-y)}$
Step 6:	$1(x + y) = 0$
Step 7:	$x + y = 0$
Step 8:	$2 + 2 = 0$
Step 9:	$4 = 0$

    Which step in Michale's proof is invalid? Why is it invalid?

24. Freddie buys an ancient Roman coin, made of bronze, with the profile of Julius Caesar on one side. On the other side, in Latin, it gives the years in which Caesar lived, 100 B.C.E to 44 B.C.E. Freddie shows the coin to his friend Jane, who says it is a fake! How does Jane know?

Assignment-Using Reasoning to Solve Problems

Name: ____________________________ Date: __________________

___
25

1. Determine the unknown term in this pattern. **What's the pattern?**

   1, 4, 16, 64, ___, 1024, 4096

2. Determine the unknown term in this pattern. **What's the pattern?**

   4, 7, 5, 8, _____, 9, 7, 10, 8

3. Determine the unknown term in this pattern. **What's the pattern?**

   2, 2, 4, 6, _____, 16, 26, 42

4. Draw the next figure in this sequence. **What's the pattern?**

Figure 1

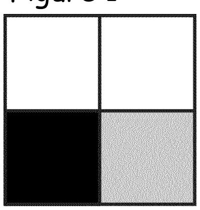


Figure 2

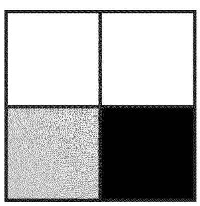
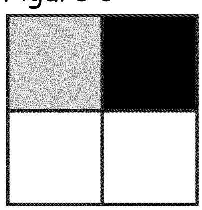


Figure 3



5. Draw the next figure in this sequence. **What's the pattern?**

Figure 1

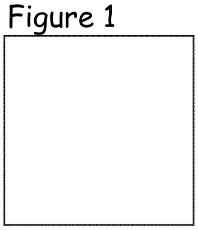


Figure 2

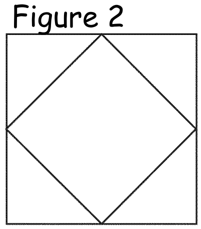
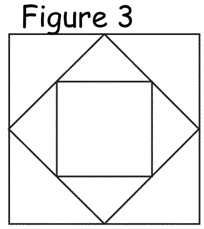


Figure 3



6. What number should appear in the centre of Figure 4? **What’s the pattern?**

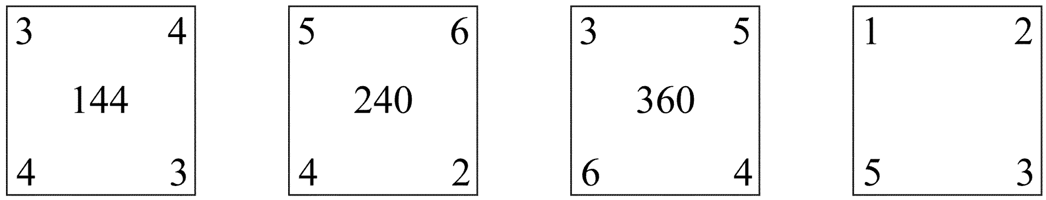


Figure 1 Figure 2 Figure 3 Figure 4

7. What number should appear in the centre of Figure 4? **What’s the pattern?**

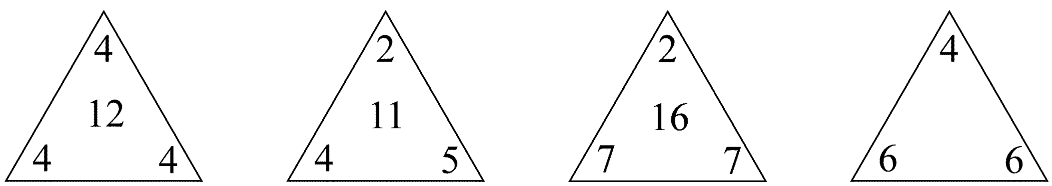


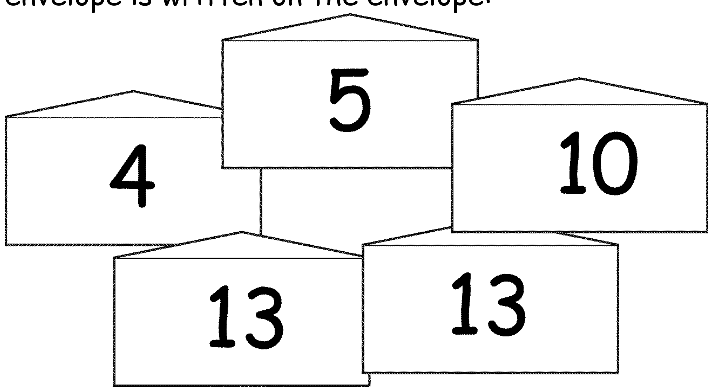
Figure 1 Figure 2 Figure 3 Figure 4

8. Andrew, Bertha, Carla, and Dixon all live on the same street. One is a chef, one is a police officer, one is an editor, and one is a travel agent. **Use the statements below to determine which person is the chef.**

   • Dixon and Carla eat dinner with the editor.
   • Andrew and Bertha carpool with chef.
   • Carla watches soccer on television with the chef and the editor.

9. Alexandra, Morana, Rebecca, and Yvonne play on the high school basketball team. After the first quarter of one game, Morana led Rebecca by 3 points. Yvonne led Alexandra by 5 points, and Rebecca led Alexandra by 2 points. In the second quarter, Alexandra got 4 points while Rebecca was scoreless. At half time, Yvonne was ahead of Morana by 4 points and Morana was 4 points ahead of Rebecca. Morana, Yvonne, and Rebecca did not play in the second half of the game. At the end of the game, Alexandra had 2 more points than Yvonne. **Who finished third in scoring?**

10. A set of 10 cards, each showing one of the digits from 0 to 9, is divided between five envelopes so that there are two cards in each envelope. The sum of the cards inside each envelope is written on the envelope:



What pair of cards is definitely in an envelope marked 13? Explain.

11. Chloe has a deck of coloured cards. One fifth of the cards are blue on both sides, and the rest have different colours on each side. Chloe laid out the cards. There were 10 blue cards, 16 yellow cards, and 4 red cards. When she flipped the cards over, she saw 12 blue cards, 10 red cards, and 8 yellow cards. **How many cards are blue and yellow?**

12. In a magic square, the columns, rows, and diagonals all add up to the same total. Use the natural numbers from 1 to 16 to complete this magic square. Use each number only once.

9			16
	15		5
14		8	
7		13	2

____ 13. Choose the next figure in this sequence.

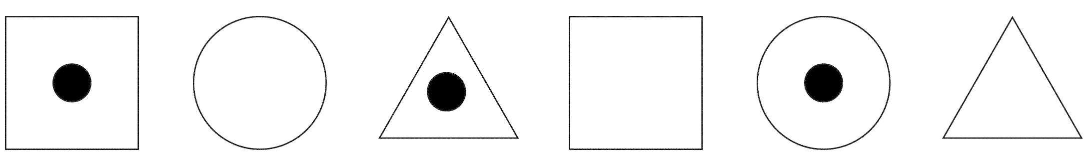
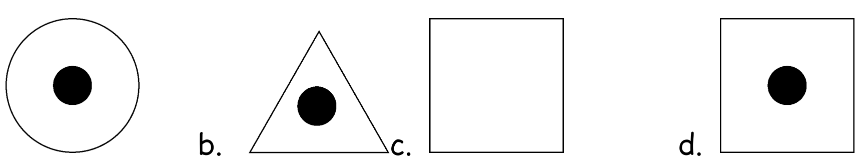


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5 Figure 6

a. b. c. d.



Conjectures
Answer Section

MULTIPLE CHOICE

1. ANS: A PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
2. ANS: C PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
3. ANS: A PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
4. ANS: D PTS: 1 DIF: Grade 11 REF: Lesson 1.2
   OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
   TOP: Validity of conjectures KEY: conjecture| validity of conjectures
5. ANS: D PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
6. ANS: B PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
7. ANS: D PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
8. ANS: C PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the

reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

9. ANS: D PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

10. ANS: C PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

SHORT ANSWER

11. ANS:
No, it is not reasonable. You need more than one example to make a conjecture.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

12. ANS:
For example, the product will be an even integer.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

13. ANS:
For example, people are more likely to buy saskatoon berry pies.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

14. ANS:
For example:

Kathryn: Want to have dinner later?
Jamie: Great! See you after work.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

Valid Conjectures
Answer Section

MULTIPLE CHOICE

1. ANS: C PTS: 1 DIF: Grade 11 REF: Lesson 1.2
   OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
   TOP: Validity of conjectures KEY: conjecture| validity of conjectures
2. ANS: D PTS: 1 DIF: Grade 11 REF: Lesson 1.2
   OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
   TOP: Validity of conjectures KEY: conjecture| validity of conjectures
3. ANS: B PTS: 1 DIF: Grade 11 REF: Lesson 1.2
   OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
   TOP: Validity of conjectures KEY: conjecture| validity of conjectures

SHORT ANSWER

4. ANS:
   Yes, it is reasonable, because $2^2 = 4$, $4^2 = 16$, $6^2 = 3^6$, and $8^2 = 64$.
   In each case, the result is an even number.

   PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning
5. ANS:
   No, it is not reasonable, because, for example, $3 + 3^2 = 3 + 9$ or 12, and 12 is even, not odd.

   PTS: 1 DIF: Grade 11 REF: Lesson 1.1
   OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
   TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive

reasoning

6. ANS:
Yes, it is reasonable, because, for example, $2^2 + 4 = 8$, $4^2 + 2 = 18$, and $6^2 + 4 = 40$.
In each case, the result is an even number.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

7. ANS:
Yes, it is reasonable, because 7(33) = **2**3**1**, 8(33) = **2**6**4**, and 9(33) = **2**9**7**.
In each case, the first and last digits form a number that is three times the original number.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

8. ANS:
No, it is not reasonable, because 19(22) = **4**1**8**, and 48 is not twice 19.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

9. ANS:
No, the conjecture is not reasonable, because 27(11) = 297 is a multiple of 27, and the sum of its digits is 18, not 9.

PTS: 1 DIF: Grade 11 REF: Lesson 1.1
OBJ: 1.1: Make conjectures by observing patterns and identifying properties, and justify the reasoning.
TOP: conjectures and Inductive Reasoning KEY: conjecture| inductive reasoning

10. ANS:
For example, I conjectured that the spaces are grey in colour, but when
I checked, it turned out that they were white.

PTS: 1 DIF: Grade 11 REF: Lesson 1.2
OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.

TOP: Validity of conjectures KEY: conjecture| validity

11. ANS:
For example, I conjectured that the figures were different sizes, but when I measured them with a ruler, it turned out that they were the same size.

PTS: 1 DIF: Grade 11 REF: Lesson 1.2
OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
TOP: Validity of conjectures KEY: conjecture| validity

12. ANS:
For example, I conjectured that figure A had the longer top side, but when I measured them with a ruler, it turned out that they were the same size.

PTS: 1 DIF: Grade 11 REF: Lesson 1.2
OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
TOP: Validity of conjectures KEY: conjecture| validity

13. ANS:
For example, based on this evidence, one might make a conjecture that the octopus had the ability to predict the future. No, this conjecture is not reasonable at all. It is just a coincidence that the octopus predicted the winners.

PTS: 1 DIF: Grade 11 REF: Lesson 1.2
OBJ: 1.2 Explain why inductive reasoning may lead to a false conjecture.
TOP: Validity of conjectures